M. Math IInd year Midterm examination 14-09-2018.
Advanced Functional analysis

Answer all the 8 questions. Each question is worth 5 points.

If you are using any result proved in the class, you need to state it correctly.

1. Let X be a locally convex real topological vector space and let $A \subset X$ be a totally bounded set. Show that the convex hull of A is also a totally bounded set.
2. Let $\delta : [0,1] \to C([0,1])^*$ be the evaluation mapping. Show that the weak*-closure of the convex hull of $\delta([0,1]) \cup -\delta([0,1])$ is a weak*-compact set in $C([0,1])^*$.
3. Let Γ be an uncountable discrete set and let $\ell^\infty(\Gamma)$ be the vector space of bounded complex-valued functions on Γ equipped with the norm $\|f\| = sup\{|f(\gamma)| : \gamma \in \Gamma\}$. Show that the weak and norm topologies are not homeomorphic .
4. Give an example of a locally convex topological vector space that is not metrizable. Give complete details of your proof.
5. Let T denote the unit circle in the complex plane. Let $\mathcal{P}$ be space of trigonometric complex polynomials in $C(T)$. Show that $\mathcal{P}$ is a space of first category.
6. Let (X, d) be a metrizable real topological vector space. Let $T : X \to R$ be a linear map such that for any $x_n \to 0$ in X, $\{T(x_n)\}$ is a bounded sequence of real numbers. Show that $ker(T)$ is a closed set.
7. Let K be a compact convex subset of a locally convex topological vector space. Let $\partial_e K$ denote the set of all extreme points of K. Let $f : K \to K$ be a continuous affine and onto map. For $k_0 \in \partial_e K$ show that $f^{-1}(k_0)$ is an extreme closed convex set.
8. Let X, Y be metrizable topological vector spaces and X is a complete metric space. Let $\Lambda : X \to Y$ be a continuous linear onto map. Show that Y is also a complete metric space.